

B.Tech (Common for all Branches) (NEP Pattern) Semester - I  
**STBSC101 - Engineering Mathematics - I**

P. Pages : 2

Time : Three Hours



**GUG/S/25/16146**

Max. Marks : 80

- Notes :
1. All questions are compulsory.
  2. All questions carry equal marks.
  3. Non programmable calculator is permitted.

1. a) If  $y = a \cos(\log x) + b \sin(\log x)$  then show that  $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$ . 8

b) Find a and b such that  $\lim_{x \rightarrow 0} \frac{x(1+a \cos x) - b \sin x}{x^3} = 1$  8

**OR**

2. a) If  $y = \left[ x + \sqrt{x^2 + 1} \right]^m$ , then prove that  $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$  8  
Hence find  $(y_n)_0$ .

b) Evaluate the following limit 8

i)  $\lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{x^2 \sin x}$

ii)  $\lim_{x \rightarrow 0} \left\{ \frac{1}{2} \left[ \frac{a}{x} + \sqrt{\frac{x}{a}} \right] \right\}^{\frac{1}{x-a}}$

3. a) If  $u = \log \left( \frac{x^4 - y^4}{x - y} \right)$ , then show that  $x^2 \cdot \frac{\partial^2 u}{\partial x^2} + 2xy \cdot \frac{\partial^2 u}{\partial x \partial y} + 2y^2 \frac{\partial^2 u}{\partial y^2} = -3$  8

b) If  $z = f(u, v)$ ,  $u = x^2 + y^2$ ,  $v = 2xy$ , then show that 8  
$$x \cdot \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 2\sqrt{u^2 - v^2} \cdot \frac{\partial z}{\partial u}$$

**OR**

4. a) If  $u = x^2 - y^2$ ,  $v = 2xy$ ,  $f(xy) = q(u, v)$  then shows that 8  
$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 4(x^2 + y^2) \cdot \left( \frac{\partial^2 \theta}{\partial u^2} + \frac{\partial^2 \theta}{\partial v^2} \right)$$

b) If  $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$ , then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$  8

5. a) Expand  $e^x \sin y$  in power of x and y, as far as terms of third degree. 8

- b) If  $u = x + y + z$ ,  $v = x^2 + y^2 + z^2$ ,  $w = x^3 + y^3 + z^3 - 3xyz$ , Then are  $u, v, w$  functionally related? If so find their relationship. 8

**OR**

6. a) Find the minimum value of  $x^2 + y^2 + z^2$ , given that  $ax + by + cz = p$ . 8

- b) If  $x + y = 2e^\theta \cos \phi$ ,  $x - y = 2ie^\theta \sin \phi$ , then verify that  $\frac{\partial(x, y)}{\partial(\theta, \phi)} \cdot \frac{\partial(\theta, \phi)}{\partial(x, y)} = 1$ . 8

7. a) Find the R.M.S. value of  $f(x) = e^x + 1$  over the range  $x=0$  to  $x=2$ . 8

- b) Show that  $\beta(m, n) = \beta(m+1, n) + \beta(m, n+1)$ . 8

**OR**

8. a) Evaluate  $\int_0^\infty \frac{x^4}{4^x} dx$  4

- b) Evaluate  $\int_0^{2a} x\sqrt{2ax - x^2} dx$  4

- c) Find the area included between the cardioids  $r = a(1 + \cos \theta)$  and  $r = 1 - \cos \theta$ . 8

9. a) Find the rank correlation from the data given below: 8

x	65	63	67	64	68	62	70	66	68	67	69	71
y	68	66	68	65	69	66	68	65	71	67	68	70

- b) Find the equation of lines of regression for the following data: 8

x	24	13	27	12	31	42	13	29	17	11
y	24	25	21	25	22	19	24	20	25	26

**OR**

10. a) Two lines of regression are given by  $5y - 8x + 17 = 0$  and  $2y - 5x + 14 = 0$ . If  $\sigma_y^2 = 16$ , find – 8

- i) the mean values of  $x$  and  $y$
- ii) coefficient of correlation between  $x$  and  $y$
- iii)  $\sigma_x^2$ .

- b) Find the coefficient of correlation and equations of regression from the following data: 8

x	3	5	6	8	9	11
y	2	3	4	6	5	8

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